RAN-2103000206020035

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T.Y.B.Sc. (Sem. VI) Examination October - 2023

Mathematics : MTH-605 - Discrete Mathematics

[Total Marks: 50

સૂચના : / Instructions

(૧) **નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **T.Y.B.Sc. (Sem. VI)**

Name of the Subject :

☛ **Mathematics : MTH-605 - Discrete Mathematics**

Subject Code No.: **2103000206020035**

Seat No.:

Student's Signature

(2) All Questions are compulsory.

(3) Figures to the right indicate marks of the corresponding question.

(4) Follow usual notations.

Q: 1 Answer the following. (Any five) **10**

1] Give an example of a Lattice which is not Distributive.

2] If S_n is the set of all positive divisors of positive integer n and D denotes the relation of divides then draw Hasse diagram of $< S_{45}, D>$.

3] Obtain the simplified Boolean expression which is equivalent to the expression $\max_0 * \max_1 * \max_2 * \max_9$.

4] In $\langle B, *, \oplus, ', 0, 1\rangle$ simplify : $(a' * b' * c') \oplus (a * b' * c) \oplus (a * b' * c')$.

5] Give circuit diagram representation of the function $f(x, y, z) = x \cdot (y + \bar{z})$

6] In a Complemented Lattice L, prove that $a' \oplus b = 1 \Rightarrow b' < a' \; \forall \; a, b \in L$

7] Give Truth table representation of the function $f(a, b, c) = abc + ab\bar{c} + a\bar{b}c$

8] In a distributive lattice $\langle L, *, \oplus\rangle$ show that:
$(a * b = a * c) \wedge (a \oplus b = a \oplus c) \Rightarrow b = c, \forall a, b, c \in L$.

Q: 2 **Answer the following. (Any Two)** **10**

1] Let $\rho(S)$ be a power set of S. Prove that $< \rho(S), \subseteq >$ is a lattice.
Is it a chain ? Justify your answer.

2] If X = {1,2,3,4,5,6,7} and the relation R = {⟨x, y⟩ / x – y is divisible by 4} define on X then Show that R is an Equivalence relation. Also Draw the sketch its graph.

3] If X = {1, 2, 3, 4} and R ={⟨l, l⟩, ⟨l, 4⟩, ⟨4, l⟩, ⟨4, 4⟩, ⟨2, 2⟩, ⟨2, 3⟩, ⟨3, 2⟩, ⟨3, 3⟩}, then write the matrix of R and sketch its graph. Is R transitive relation? Justify your answer.

Q: 3 **Answer the following. (Any Two)** **10**

1] State and prove De-Morgan's laws in a Complemented distributive Lattice.

2] Let $< L, \leq >$ be a lattice. For $a, b, c, d \in L$, Prove that

(i) $b \leq c \Rightarrow a \oplus b \leq a \oplus c$

(ii) $a \leq b, c \leq d \Rightarrow a * c \leq b * d$

3] Prove that the direct product of two distributive lattices is distributive.

Q: 4 **Answer the following. (Any Two)** **10**

1] Show that every chain is a distributive Lattice.

2] Obtain the product of sums canonical form of $x_{1\oplus}(x_2 * x_3')$

3] Let ⟨B, ∗, ⊕, ′, 0, 1) is a Boolean algebra and let S be a nonempty subset of B.
If S preserves the operations ∗ and ′ then prove that ⟨S, ∗, ⊕, ′, 0, 1⟩ is a Sub-Boolean Algebra.

Q: 5 **Answer the following. (Any Two)** **10**

1] Use the Karnaugh map representation to find the minimize sum of product of the function ∑ (0, 2, 6, 7, 8, 9, 13, 15)..

2] Use the QuineMaCluskey algorithm to find the minimal expression of the Boolean function ∑ (0, 1, 4, 5, 9, 11)

3] Give cubic and Karnaugh map representation of the Boolean function
f(a, b, c,) = ∑(0, 2, 3, 7).

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